Catastrophe by Design in Population Games

Destabilizing Wasteful Locked-in Technologies

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Phase Transitions in Societal Systems

Can we flip social behaviour?

Motivation

The adoption of innovative, socially-optimal technologies is usually hindered or stopped by wasteful lock-ins.

Various technological dilemmas over the years:

- TCP/IP vs. ATM
- IPv4 vs. IPv6
- Conventional vs. Electric Vehicles
- Proof-of-Work (PoW) vs. Proof-of-Stake (PoS)
- etc.

Bifurcation and Catastrophe Theory finds application in these cases.

Model I: Agents and Strategies

A population of agents

- Mass $K > 0$: total population size.
- Strategies: two available technologies, W (wasteful), and S (socially-optimal).
- Inherent cost: $\gamma > 0$ for W and 0 for S.
- Population states: $X = \{(x, 1-x) : x \in [0,1]\}$ where $x =$ fraction of adopters of the wasteful technology.

Model II: Value and Payoffs

Each technology creates value split among adopters

- Value V, Growth $\alpha > 1$:
  - $V_W = V(xK)^\alpha$ and $V_S = V((1-x)K)^\alpha$
- Payoff functions: equal share amongst all participating agents:
  - $u(W, x) = V_W \cdot (xK)^{-1} - \gamma = VK^{\alpha-1}x^{\alpha-1} - \gamma$
  - $u(S, x) = V_S \cdot ((1-x)K)^{-1} = VK^{\alpha-1}(1-x)^{\alpha-1}$
- For the purposes of this talk we restrict ourselves to the case $\alpha = 2$:
  - $u(W, x) = VKx - \gamma$
  - $u(S, x) = VK(1-x)$

An Evolutionary Game

Evolutionary game interpretation

$$P = \begin{array}{c} \\ W \\ S \end{array} \begin{array}{c} \begin{array}{cc} W & S \end{array} \\ \begin{pmatrix} VK - \gamma & -\gamma \\ 0 & VK \end{pmatrix} \end{array} \tag{G1}$$

Theorem

(G1) has three equilibria: (S) and (W) the monomorphic/pure states where everyone chooses S and W, respectively, and one mixed. The two pure equilibria are evolutionary stable, whereas the mixed one is unstable.

Population Dynamics

Q-Learning dynamics:

$$\dot{x} = x[\underbrace{u(W,x) - \bar{u}(x)}_{\text{Replicator Dynamics}} - T \cdot \underbrace{(x \ln x + (1-x)\ln(1-x))}_{\text{Entropy}}]$$

Where $\bar{u}(x) = xu(W,x) + (1-x)u(S,x)$

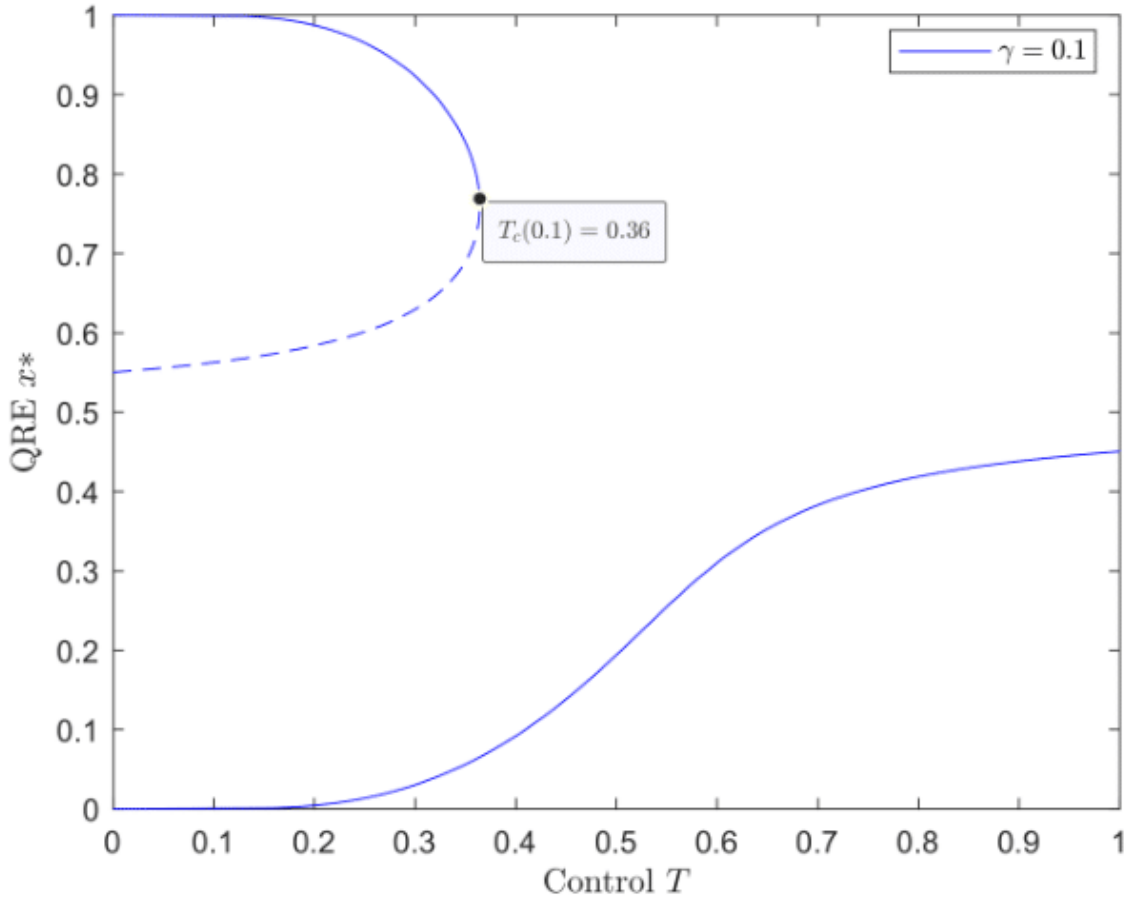
Quantal Response Equilibrium (QRE): The steady states of the system, i.e., $\dot{x} = 0$.

We can affect the agents' rationality by scaling the agents' utilities:

$$x[\frac{u(W,x)}{c} - \frac{\bar{u}(x)}{c} - T \cdot (x \ln x + (1-x)\ln(1-x))] = 0$$
$$\iff x[u(W,x) - \bar{u}(x) - cT \cdot (x \ln x + (1-x)\ln(1-x))] = 0$$

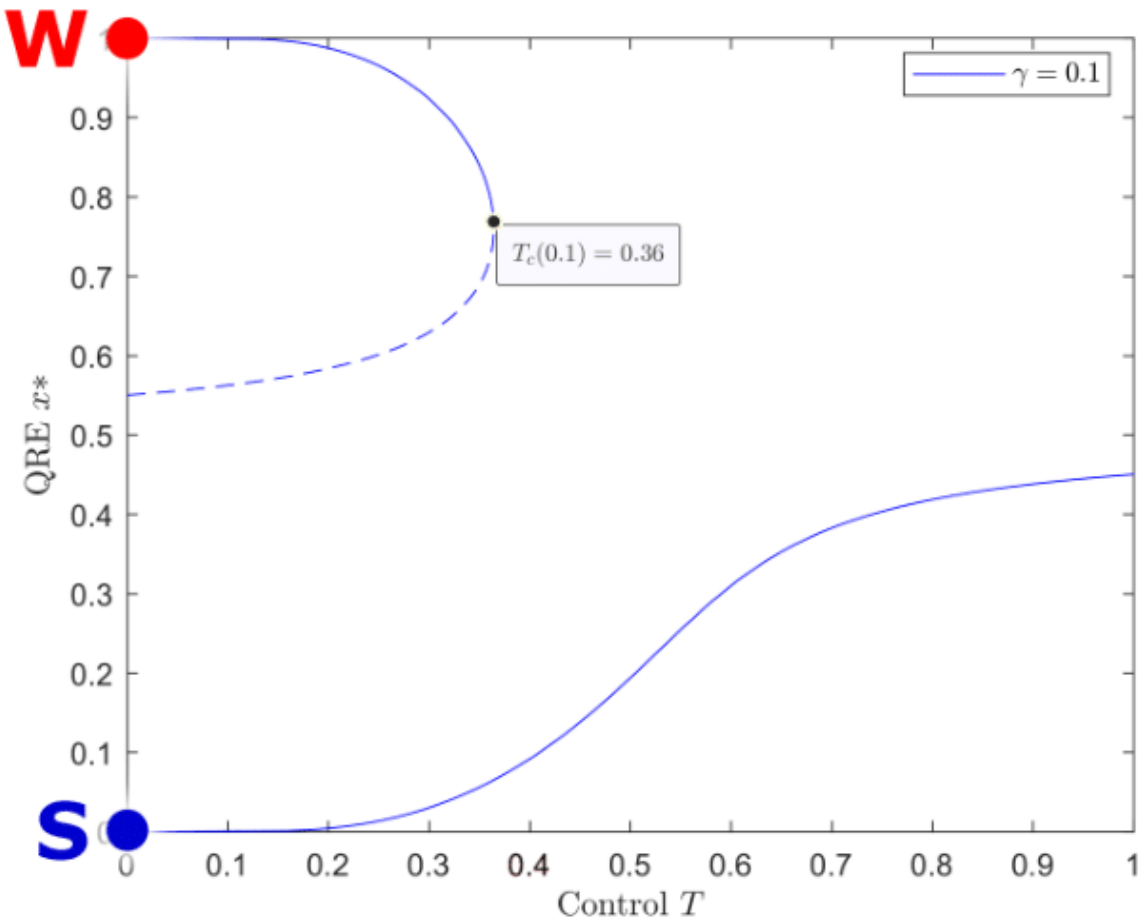
QRE Correspondence: Visually

In our case ($\alpha = 2$): $\dot{x} = x(1-x)[2x - (1+\gamma) - T\ln(\frac{x}{1-x})]$



Can we Flip the Population's Behaviour?

Can we flip the system from the bad equilibrium to the good one?



QRE Correspondence: Formally

Theorem

For any $\alpha > 1$ there exists a finite sequence of temperatures $T =< T_0, T_1, \cdots >$ such as starting from an initial state x_0 and performing the following procedure for each $T_i \in T$:

- *Scale the system's temperature at T_i, and*
- *Wait until the system converges to a QRE*

the system is going to converge to the desirable state $x = 0$ which corresponds to socially-optimal technology S.

We can reliably destabilize a wasteful equilibrium and converge to a socially-optimal equilibrium by introducing and removing taxes in the system.

Short Term Policy $\Longrightarrow$ **Long Lasting Effects**

QRE Correspondence: Proof I

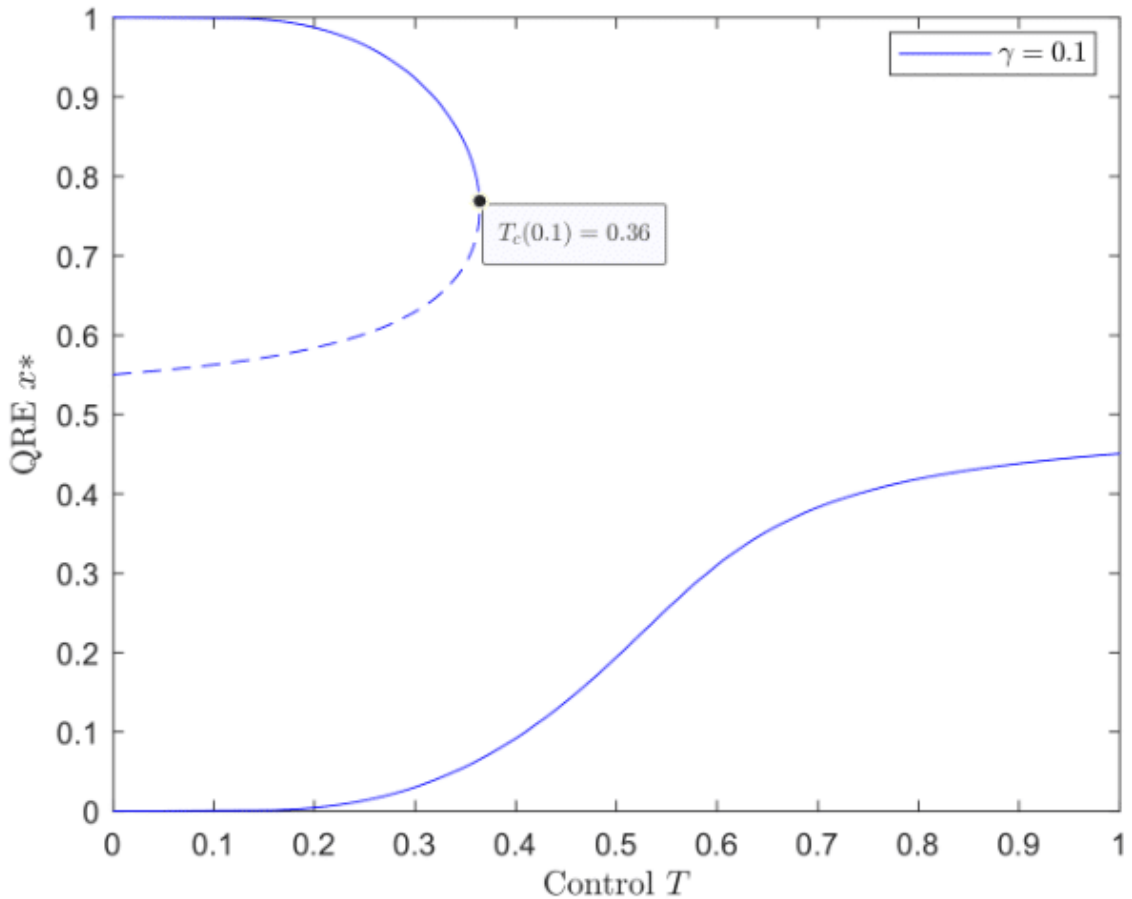
Sketch of the proof:

1. Show that for every fixed temperature $T \geq 0$, the system always converges to equilibrium points.
2. Design a sequence of temperatures such that as we move through the sequence of self-stabilizing systems the final system converges to the socially-optimal equilibrium.

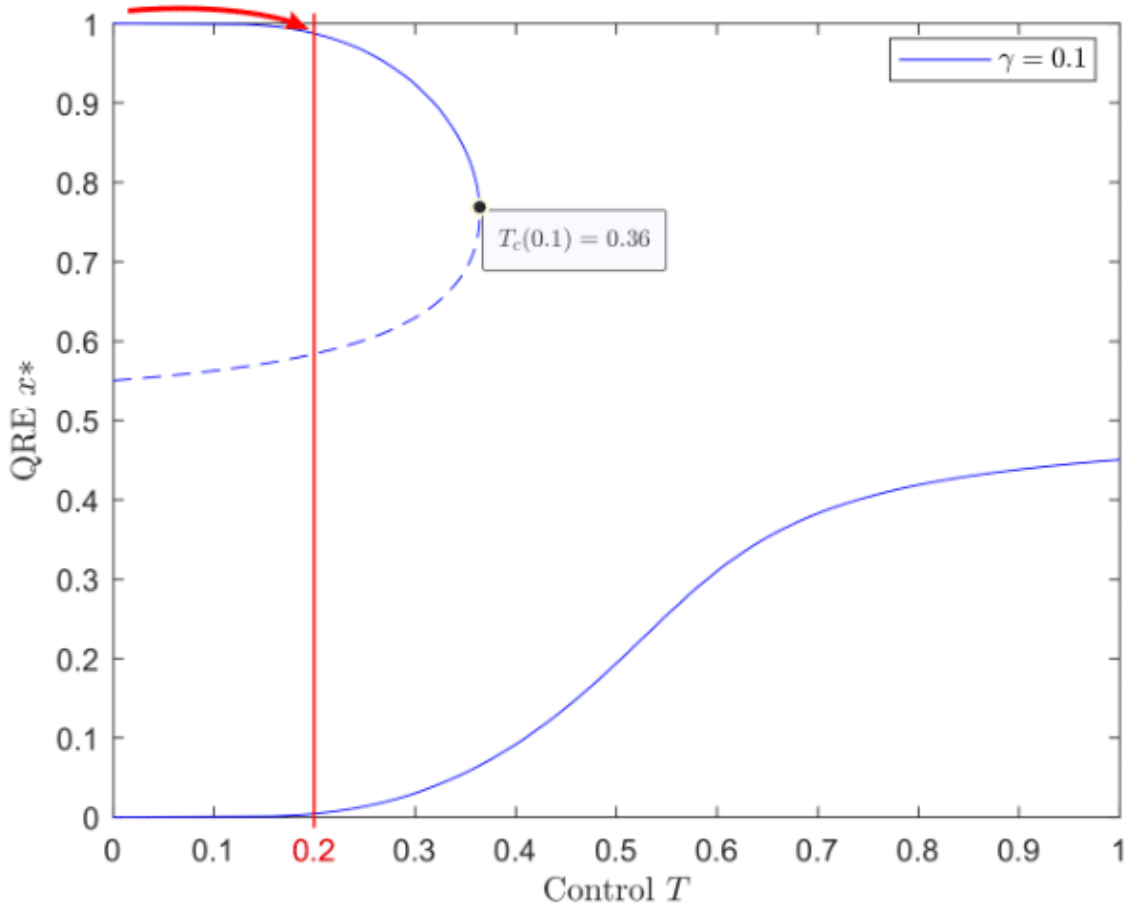
Step 1 is proved via Lyapunov Theory.

Step 2 is proved via Catastrophe or Bifurcation Theory.

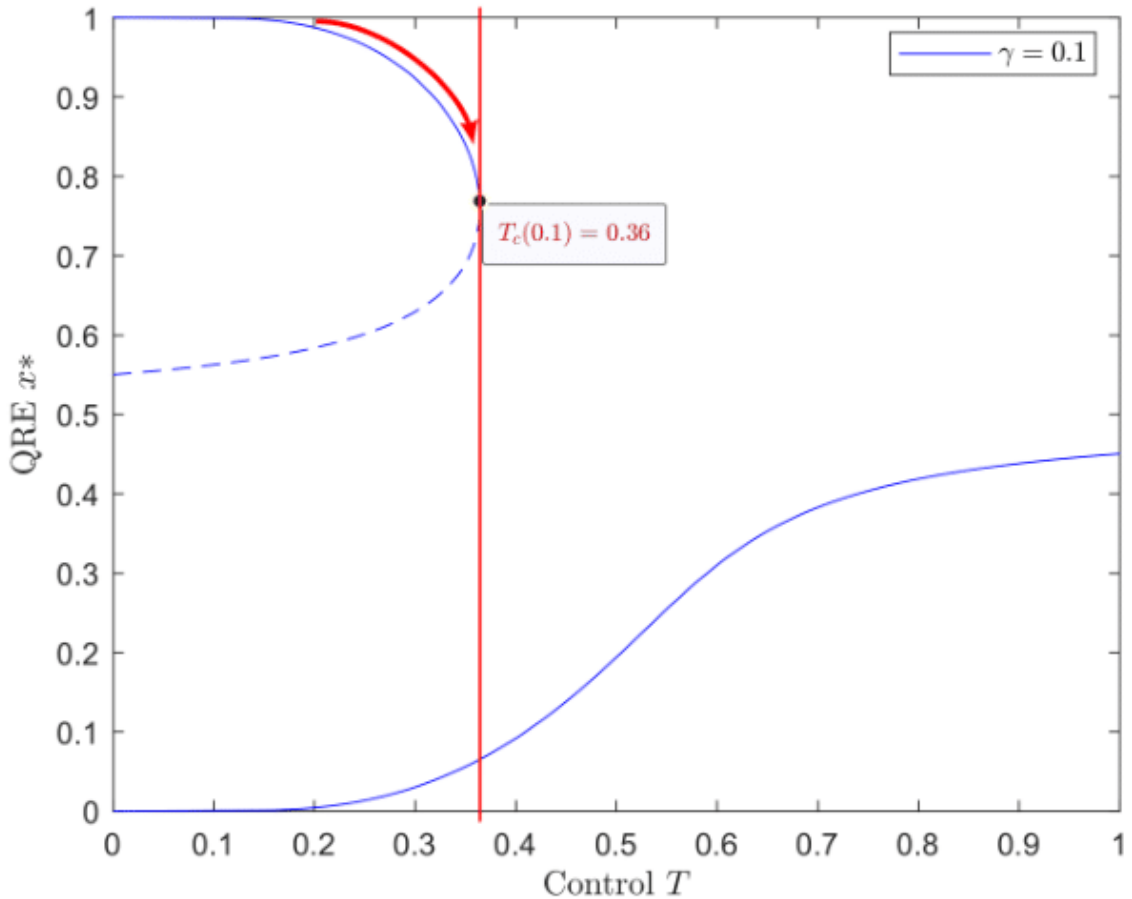
QRE Correspondence: Proof II



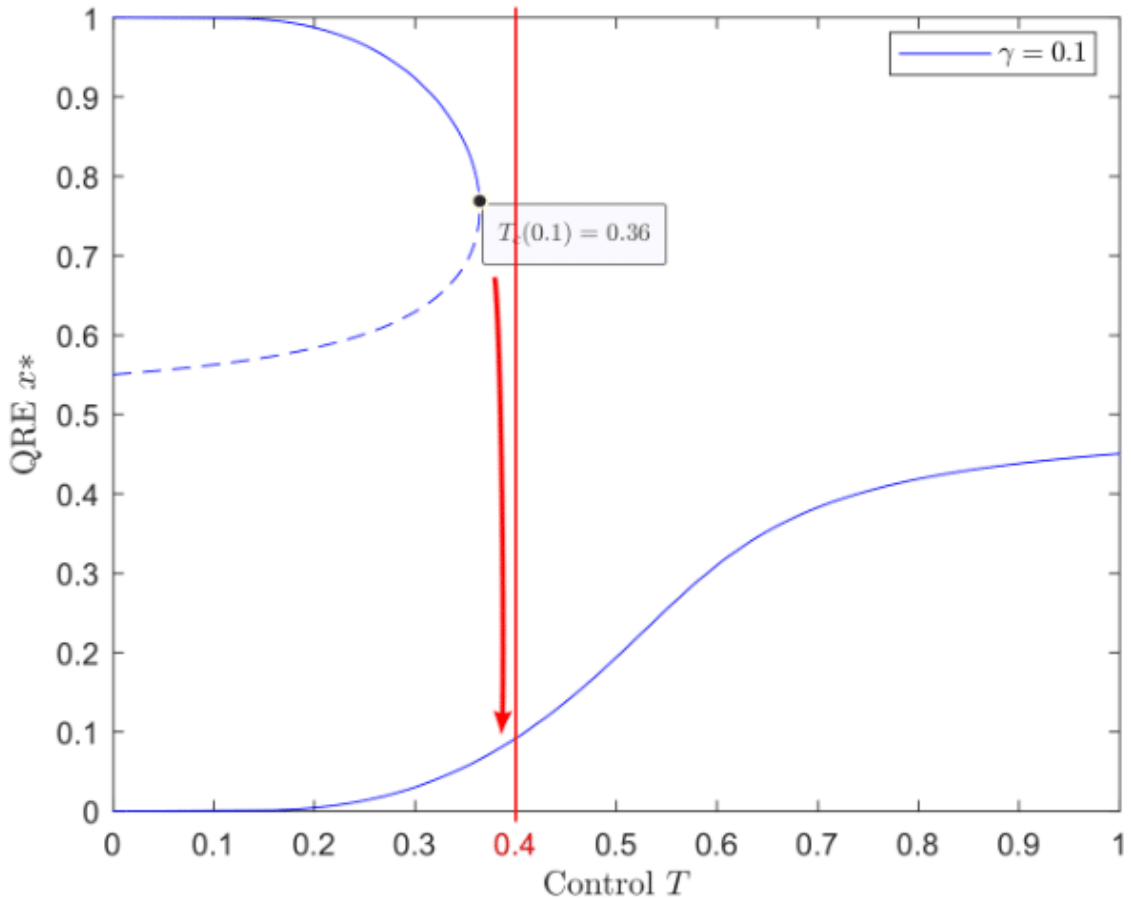
QRE Correspondence: Phase 1



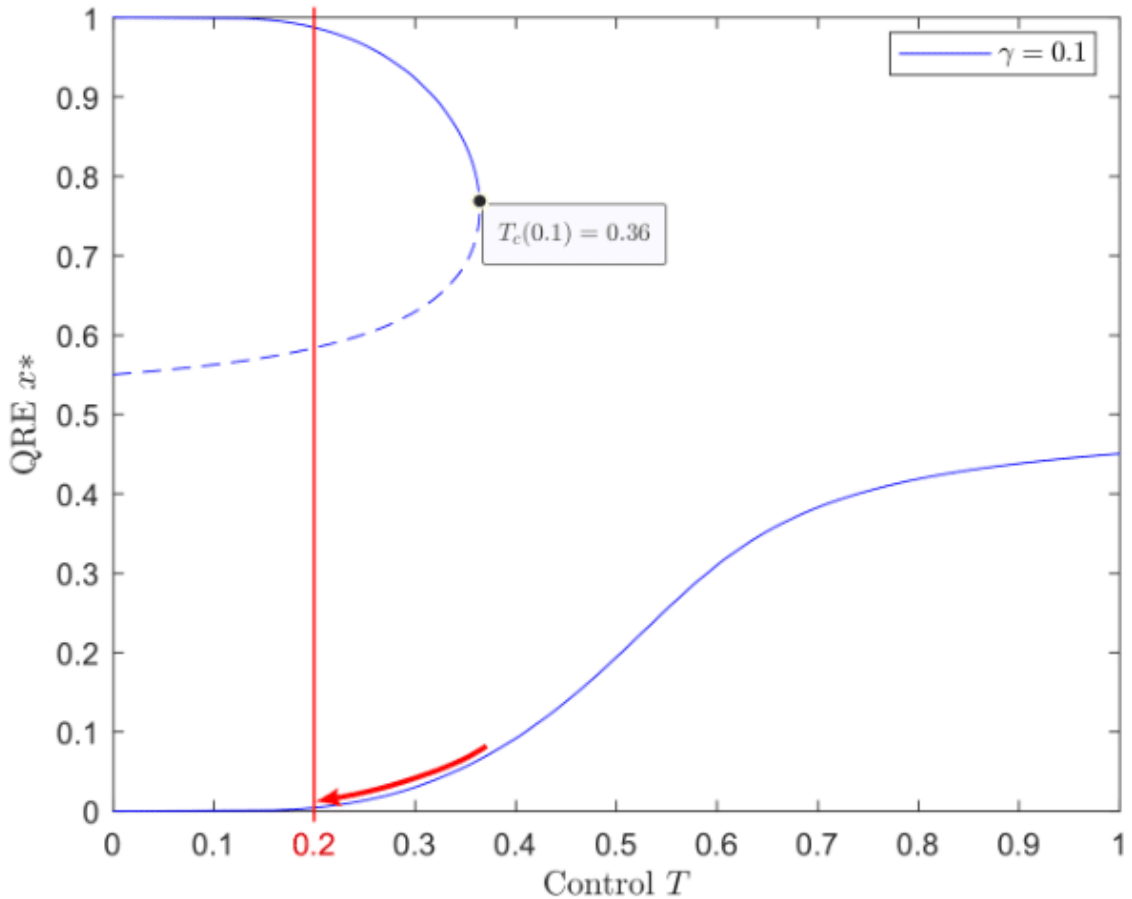
QRE Correspondence: Phase 2



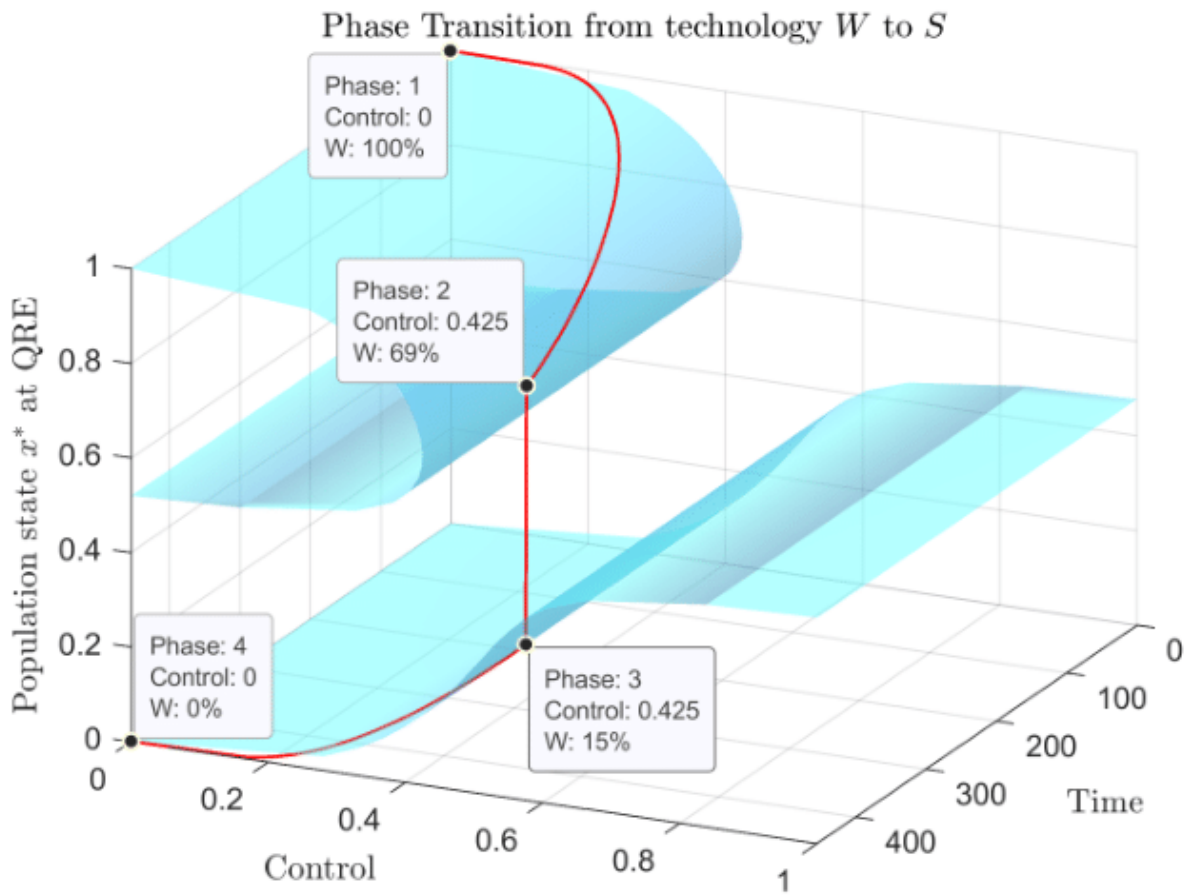
QRE Correspondence: Phase 3



QRE Correspondence: Phase 4



Designing a Catastrophe



Conclusion

Can we flip social behaviour?

Yes, we can!

HOW? CATASTROPHE BY DESIGN

- *Destabilize* ***bad equilibrium***
- *Induce* ***bifurcation*** *(# Equilibria change)*
- ***Phase transition*** *(jump to another branch of equilibria)*
- *Remove control to stabilize* ***good equilibrium***